**南京大学数学系2017年暑期学校安排**

2017年南京大学数学系暑期学校活动安排如下：

**时间：**7/12-7/30（共三周）

**地点：**南京大学鼓楼校区（南京市汉口路22号）

**听课对象：三、四年级本科生及低年级研究生**

**内容：**

1. 短课（共五门，每门讲授5-12次，每次2-3课时不等）
2. 椭圆偏微分方程（关波）
3. 代数几何 (王博潼)
4. 黎曼几何（梅加强）
5. 渐进双曲爱因斯坦流形与共形几何（庆杰）
6. Galois表示（Ravi Ramakrishna）

二．专家讲座：若干

欢迎兄弟院校学生参加, 参加活动的学生可直接向数学系报名, 报名时请注明计划参加的课程名称, 活动主办方收取材料费用100元, 安排食宿。

**报名截止日期**：2017年6月20日

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**报到日期**：2017年7月11日

短课授课计划：

**一、椭圆偏微分方程（关波，俄亥俄州立大学）（18课时）**

1. 教学目的和内容

The course is roughly divided into two parts. The first part covers some of the fundamental topics in the theory of linear equations, including the maximum principle and the Harnack inequality of Krylov-Safanov. The second part concerns fully nonlinear elliptic equations. We shall discuss the Evans-Krylov estimates, and the Dirichlet problem for the Monge-Ampere as well as more general Hessian equations.

1. 教学进度和具体内容安排（预先安排，可能有调整）

Chapter 1. Linear uniformly elliptic equations

1.1 The maximum principle

1.2 A priori estimates

1.3 The Alexandrov maximal principle

1.4 Krylov-Safanov's Harnack Inequality.

1.5 The Holder estimates.

Chapter 2. Fully nonlinear elliptic equations.

2.1 Evans-Krylov theory

2.2 The Monge-Ampere equation

2.3 Hessian equations

1. 参考书籍
2. 韩青: “Nonlinear elliptic equations of the second order”, AMS, 2016.

**二、代数几何（王博潼，威斯康辛大学）（10课时）**

1. 教学目标和内容：

 Algebraic geometry studies the solution space of a system of polynomial equations. The modern approach to algebraic geometry is via scheme theory, which highly rely on the theory of commutative rings. However, when the coefficients of the polynomials are complex numbers, we can study the solution spaces by analytic method. In this course, I will give an introduction to complex algebraic geometry with emphasis on analytic method. The main topics we will cover are, complex projective varieties, complex manifolds, singularities, Riemann surfaces, sheaf theory.

1. 教学进度和具体内容安排：

1. A brief introduction to algebraic geometry. What is algebraic geometry? Why do people study algebraic geometry? What are the important questions in algebraic geometry?

2. Holomorphic functions, real and complex manifolds, analytic varieties.

3. Sheaf theory.

4. Affine varieties, Zariski topology and abstract algebraic varieties.

5. Riemann surfaces and algebraic curves.

6. Bezout's theorem.

1. 参考文献

1）Claire Voisin: Hodge theory and complex algebraic geometry. Volume 1

2）Done Arapura: Algebraic geometry over the complex numbers.

3）Phillip Griffiths and Joseph Harris: Principles of algebraic geometry.

4）R. O. Wells: differential analysis on complex manifolds.

**三、黎曼几何（梅加强，南京大学）（16课时）**

1. 教学目标和内容:

本课程介绍现代黎曼几何的基本知识，主要讲授测地线、指数映射、Jacobi场等基本对象和研究工具，并适当地给出几个应用，包括Hopf-Rinow定理，Synge定理，Bonnet-Myers定理等。

2.教学进度和具体内容安排:

1）黎曼几何的基本概念

2）测地线和指数映射

3）黎曼流形的完备性

4）Jacobi场和弧长的变分公式

5）二次变分公式及其应用

6）割点与割迹

7）比较定理及其应用

3.参考文献：

（1）伍鸿熙等，黎曼几何初步，北京大学出版社，2000

（2）曹建国，现代黎曼几何简明教程，科学出版社，2006

（3）Petersen，Riemannian Geometry, Spinger, 2006

**四、渐近双曲爱因斯坦流形和共形几何（庆杰， 加州大学）（18课时）**

1. 教学目标和内容:

In these lectures, we will introduce the research on asymptotically hyperbolic (AH) Einstein manifolds and the uses of AH Einstein manifolds in the study of conformal geometry, in the spirit of the so-called AdS/CFT correspondence.  
 We will start with the ambient space construction of Fefferman and Graham to motivate the mathematics behind the so-called AdS/CFT correspondence.We will discuss some analysis on AH manifolds and the perturbational existence theorem of Graham-Lee and Lee.We will study how the geometry of an AH Einstein manifold and its conformal infinity are related, and derive curvature estimates for AH Einstein manifolds.

2.教学进度和具体内容安排:

1) Rigidity  
2) Renormalized volume and topology of AH Einstein 4-manifolds  
3) Existence and compactness  
4) Kleinain manifolds and scattering operators  
5) Hypersurfaces in hyperbolic space

3.参考文献：

庆杰:“Conformal geometry and partial differential equations” (Personal Lectures) 2017.

**五、伽罗华表示（Ravi Ramakrishna， 康奈尔大学）（16课时）**

1. 教学目标和内容:

 This is the branch of mathematics concerned with symmetries of solutions of equations. There is an object that encodes all symmetries of solutions to all equations, the absolute Galois group of the rational numbers. I study this object and its relations with number theory. The study of these symmetries has gained an increasingly important role in number theory in recent years. In particular, Galois theory played an important role in the solution of Fermat's Last Theorem.

2.教学进度和具体内容安排:

待定

3.参考文献：

待定

**课表：**

第一周（7/12-7/18）

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | 7/12 | 7/13 | 7/14 | 7/15 | 7/16 | 7/17 | 7/18 |
| 上午1-2节 | 共形几何 | 共形几何 | 共形几何 | 专家讲座日 | 休息日 |  | 椭圆偏微分方程 |
| 上午3-4节 | Galios 表示 | Galois表示 | Galois表示 |  |  | Galois表示 | Galois表示 |
| 下午5-7节 | 椭圆偏微分方程 | 椭圆偏微分方程 | 椭圆偏微分方程 |  |  | 椭圆偏微分方程 | 共形几何 |

第二周（7/19-7/25）

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | 7/19 | 7/20 | 7/21 | 7/22 | 7/23 | 7/24 | 7/25 |
| 上午1-2节 | 椭圆偏微分方程 | 椭圆偏微分方程 | 代数几何 | 专家讲座日 | 休息日 | Galois表示 |  |
| 上午3-4节 | Galois表示 | Galois表示 | 黎曼几何 |  |  | 代数几何 | 代数几何 |
| 下午5-7节 | 共形几何 | 共形几何 | 共形几何 |  |  | 黎曼几何 | 黎曼几何 |

第三周（7/26-7/30）

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | 7/26 | 7/27 | 7/28 | 7/29 | 7/30 |
| 上午1-2节 | 代数几何 | 代数几何 |  | 专家讲座日 | 休息日、结束日 |
| 上午3-4节 |  |  | 黎曼几何 |  |  |
| 下午5-7节 | 黎曼几何 | 黎曼几何 |  |  |  |